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ABSTRACT

This study was designed to examine the processes underlying developmental changes in children's (1) use of combinatorial strategy, and (2) comprehension of conjunctive and disjunctive propositional relationships. A total of 108 children from third, sixth and eighth grades participated in this study. Each child was administered three tasks (traditional, disjunctive, and conjunctive) and received either the traditional univariate colored token described by Inhelder and Paiget, or one of two bivariate tasks. In the traditional task situation the child was presented with multiple instances of half-circle shapes of six different colors arranged in a row, and was instructed to make all the different pairs of colors possible, using two different colors in each pair. In the disjunctive task, multiple instances of red, blue, and yellow squares and circles were arranged in a row and the child was instructed to make all the different possible pairs, using either a different color or a different shape in each pair. In the conjunctive task, each child was presented with the same stimuli as in the disjunctive task but this time was instructed to make all the different possible pairs using a different color and a different shape in each pair. Performance criteria based on descriptions by Inhelder and Piaget of the behavior characteristics of each substage of development on the colored tokens task were used to rate each child's use of a combinatorial strategy. Results suggest that (1) use of a combinatorial strategy is significantly affected by the dimensional complexity of the task, and (2) comprehension of disjunctive and conjunctive propositional relationships does not differ in children within the age range included in this study. (Author/MP)

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Logical Structures Underlying Combinatorial Reasoning

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Piaget's theory asserts that formal operational thought derives from a structure characterized by the combinatorial system and propositional logic. The present study was designed to examine the processes underlying developmental changes in 1) Use of a combinatorial strategy, and 2) Comprehension of the propositional relationships implied by the language connectives "and" and "or".

Several different tasks - colored tokens, permutations, arrangements, and chemicals - have been used to assess use of the combinatorial strategy and to infer presence of the combinatorial system, and previous research has indicated that the colored tokens task elicits a formal operational performance at an earlier age than do the other tasks (Martorano, 1977). Comparison of these tasks suggests that while the colored tokens task requires the child to produce all possible pairs of univariate stimuli (e.g., a red square paired with a green square), the remaining tasks require application of a combinatorial strategy with bivariate stimuli. That is, the arrangements and permutations tasks are based on both the identity of the stimulus and its position in the array (i.e., 1234, 2134); the chemicals task requires combinations based on both the identity of the chemical and the level of chemical interaction (i.e., 1-way, 2-way, etc.). We hypothesized that one reason for the ease of the colored tokens task was the presence of univariate stimuli. Thus, performance on the traditional task was compared to performance on two bivariate tasks.

A second focus of the study was the description of the processes underlying children's comprehension of conjunctive and disjunctive propositional relationships. Previous research has presented verbal statements expressing conjunctive and disjunctive relationships and asked the child to either identify an attribute combination as true or false with respect to the statement, or circle correct attribute instances presented within a larger array, e.g., all things which are square and black would imply that all black squares should be circled. These studies have indicated that while conjunctive relationships are readily understood by children within a concrete age range, disjunctive relationships are not well understood until late adolescence (Neimark & Slotnick, 1970). However, each of these studies relied on the child's accurate identification of instances of the verbal propositional statement as an indication of comprehension of the propositional relationship. We were interested in whether the same differential difficulty in conjuncitve and disjunctive relationships would occur when children were asked to actually construct rather than just identify instances of each propositional relationship; a method which would permit us to examine the strategies used by each child when constructing the attribute combinations.

Method

Subjects. 108 children from the 3rd, 6th, and 8th grades participated in this study.

Tasks. Each child received either the traditional univariate colored tokens task described by Inhelder and Piaget, or one of two bivariate tasks. In the traditional task situation the child was presented with multiple instances of half-circle shapes of 6 different colors (red, blue,

green, yellow, orange, and black), layed out in a row (e.g., a pile of reds, etc.) and was instructued to "make all the different pairs of colors possible, using two different colors in each pair, and being sure not to repeat any of the pairs.

In the disjunctive task, multiple instances of red, blue, and yellow squares and circles were arranged in a row and the child was instructed to, "make all the different possible pairs, using either a different color Or a different shape in each pair and making sure not to repeat any of the pairs."

In the conjunctive task, the stimuli once again included multiple instance of red, blue and yellow squares and circles arranged in a row. However, the child was instructed to, "make all the different possible pairs using a different color AND a different shape in each pair, and being sure not to repeat any pairs."

Although the traditional task and the disjunctive task differed with respect to number of dimensions represented in the stimuli, both tasks allowed for the use of an identical strategy for the systematic and exhaustive construction of pairs. This strategy, defined by Piaget as evidence of the combinatorial system, involves pairing all stimuli with a single stimulus (e.g., red with green, red with blue, etc.) followed by pairing all stimuli with a second stimulus and repetition of this strategy until all pairs have been made. Thus, although dimensional complexity varied between the traditional and the disjunctive tasks, the optimal strategy for task solution and the number of pairs which could possibly be constructed (15) was the same.

In contrast, the conjunctive and disjunctive tasks represented identical dimensional complexity, but varied with respect to number of possible pairs (15 for disjunctive, 6 for conjunctive), and the task definitions of a legitimate pair reflected different logical propositional relationships.

Procedure. Each child was individually tested and, prior to presentation of the test stimuli, observed the interviewer model construction of pairs with a simplified version of the task involving multiple instances of three stimuli which differed in color and shape from the test stimuli. During the modeling task, the interviewer verbalized the appropriate task instructions (i.e., traditional, disjunctive, conjunctive) and then constructed legitimate pairs and verbalized why they were correct ("I can put a purple triangle with a pink diamond because they are a different shape and a different color"). On the disjunctive task, the model constructed a pair representing an inclusive instance of the disjunctive relationship. On all three tasks the interviewer constructed a repeated pair and verbally explained why this was a repeat and why it was not legitimate and then took the pair away. Construction of the pairs did not follow any systematic strategy, and once all possible pairs of the three stimuli had been made the interviewer modeled checking for repeated or omitted pairs. This pretest modeling served to visually and verbally instruct the child about the task requirements - constructing all possible pairs with no repeats, and to provide correct instances of the propositional relationship which defined a legitimate pair.

Following the pretest modeling of the task instructions, the interviewer placed the multiple instance of the actual task simuli in a row in front of the child and verbalized the appropriate task instructions. The interviewer then recorded the order and identity of each pair constructed by the child. When the child indicated that (s)he had finished, the interviewer replaced the task stimuli in the original row arrangement and asked the child to make all the possible pairs a second time.

Scoring

Piagetian Scoring. Performance criteria based on descriptions by Inhelder and Piaget (1958) of the behavior characteristics of each substage of development on the colored tokens task were used to rate each child's use of a combinatorial strategy. The criteria, ranging from 1 indicating a preoperational level of performance to 5 indicating a late formal operational level of performance, were:

1 Preoperational: random and haphazard pairing of stimuli

2 Early concrete: sequence of pair construction reflects perceptual chaining

3 Late concrete: Sequence of pair construction reflects initial use of system which is dropped

4 Early Formal: A system is used exhaustively, but repeated or omitted pairs are present

5 Late Formal: A system is carried through to completion without repeats or omissions.

Results

A preliminary ANOVA indicated no significant $T_1 - T_2$ difference and therefore scores were averaged across the two trials. The ANOVA of the Piagetian scores revealed a significant increase with grade in level of performance, F (2, 90) = 34.917, p .001, and Newman-Keuls post hoc comparisons indicated significant differences between each grade. The task main effect was also significant, F (2,90) = 8.097, $p < .001$, and a Newman-Keuls post hoc comparison revealed that performance on the traditional task was significantly higher than performance on either the conjunctive or the disjunctive tasks. There were no significant differences in level of performance between the conjunctive and disjunctive tasks. Table 1 on your handout presents the mean Piagetian scores and standard deviations for each grade and task group. Inspection of the table reveals that the mean level of performance by 3rd grade children was at an early concrete operational level on all tasks, and that only on the traditional colored tokens task was mean performance close to an early formal operational level. However, mean performance of the 8th grade children was at a late formal level on the colored tokens task, and at an early formal level on the disjunctive and conjunctive tasks.

Examination of the number of children at each grade, on each task, who were scored as either early or late formal operational indicated that on the traditional task the majority of both 6th and 8th grade children evidenced a formal strategy while only a majority of the 8th grade

children were rated as formal operational in their performance on the disjunctive and conjunctive tasks. Virtually none of the children in any of the grades made repeated pairs or logically illegitimate pairs on any of the tasks. However, there were significant age and task main effects in the proportion of omitted pairs. The ANOVA of these data revealed that there was a significant decline with grade in the proportion of omitted pairs. Post hoc analyses indicated that the significant decline occurred between the 3rd and 6th grades. With respect to the task main effect, significantly more omitted pairs occurred on the disjunctive than on either the conjunctive or the traditional colored tokens tasks. These data are presented in table 2 of the handout.

In an effort to examine the source of children's omission errors on the disjunctive task, each child's omitted pairs were classified as being either a between class omission in which both color and shape differed, or a within class omission in which either color or shape differed. This analysis revealed that while the 3rd grade children had more within than between class omissions, and the 6th grade children's omissions were equally divided between within and between class omissions, the 8th grade children had more between than within class omissions.

Discussion

The results of the present study suggest that one source of the relatively high level of performance on the colored tokens task, in contrast to other combinatorial tasks, is dimensional complexity. At each grade, children utilized a more systematic combinatorial strategy,

in terms of the Piagetian criteria, on the univariate traditional task, than they did on the bivariate disjunctive and conjunctive tasks. The equivalent levels of performance on the conjunctive and disjunctive tasks, with respect to use of a systematic combinatorial strategy, also provides evidence that developmental differences in use of the combinatorial strategy are due to developmental changes in the child's ability to coordinate multidimensional information. Furthermore, the very low frequency of repeated pairs suggests that failure to use a combinatorial strategy was not due to inattention to the dimensional attributes of the stimuli.

Although the conjunctive and disjunctive tasks did not elicit differential levels of the combinatorial strategy, the analyses of the omission data did reveal differences between the two tasks. Specifically, on the disjunctive task proportionally more of the possible logically legitimate pairs were omitted. Furthermore, the 3rd grade children omitted more within class (e.g., either color or shape) pairs, while the 8th grade children omitted more between class (both a different color and shape) pairs. However, since the absolute frequency of omitted pairs was extremely low we would argue that children within the age range included in this study do not differ in their comprehension of conjunctive and disjunctive propositional relationships.

The discrepancy between the results of the present study and those of previous studies showing performance differences on tests of conjunctive and disjunctive relationships may be due, as Neimark and Slotnik (1970) have suggested, to the greater complexity of cognitive operations necessary

for the child to apply his or her knowledge of the disjunctive relationship. In our study children constructed rather than identified stimuli representing instances of a particular propositional relationship. Consequently, the child was forced to actively process information from both dimensions for each stimulus member of the constructed pair. In contrast, the recognition method may have failed to elicit complete processing of the stimulus information and thus led to lower performance on disjunctive statements.

In summary, the results of the present study suggest that 1) Use of a combinatorial strategy is significantly affected by the dimensional complexity of the task, and 2) Comprehension of disjunctive and conjunctive propositional relationships does not differ in children within the age range included in this study.

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LOGICAL STRUCTURES UNDERLYING COMBINATORIAL REASONING

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Table 1

Mean Piagetian Scores and Standard Deviations

for each Combinatorial Task at each Grade Level

	Task			
	Traditional	Disjunctive	Conjunctive	Mean
Grade				
3				
$\bar{X}$	2.38	1.58	2.04	2.0
SD	1.45	.90	1.10	1.18
6				
$\bar{X}$	3.96	2.63	2.75	3.11
SD	.99	1.09	.89	1.14
8				
$\bar{X}$	4.58	3.92	3.75	4.08
SD	.56	1.04	1.29	1.05
Mean				
$\bar{X}$	3.64	2.71	2.85	3.07
SD	1.40	1.38	1.29	

Note. Maximum Score = 5.0

Table 2

Mean Proportion Omitted Pairs for each Combinatorial Task at each Grade Level

	Task			
	Traditional[a]	Disjunctive[b]	Conjunctive[c]	Mean
Grade				
3				
$\bar{X}$	.306 (2)[d]	.433 (6.5)	.319 (2)	.353 (4.3)
SD	.32	.24	.24	.27
6				
$\bar{X}$	.039 (0.5)	.225 (3.5)	.118 (1)	.127 (1.7)
SD	.06	.20	.18	.17
8				
$\bar{X}$	.006 (-)	.103 (1.5)	.013 (-)	.040 (0.5)
SD	.02	.17	.19	.15
Mean				
$\bar{X}$	.117 (2)	.254 (4)	.150 (1)	(2.3)
SD				

Note.

$$\text{Proportion of Omissions} = \frac{\text{\# of omitted pairs}}{\text{\# of possible pairs}}$$

[a] Out of 15 pairs

[b] Out of 15 pairs

[c] Out of 6 pairs

[d] Mean number omitted pairs